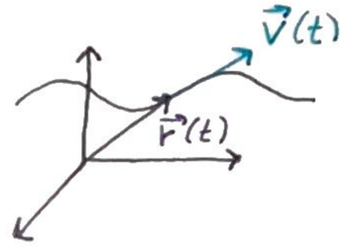
Mekanik-föreläsning 4.3.1 Carlos 12/4-18

Kapitel 7 - kinematik

Kraft $\leftrightarrow$ Rörelse $\rightarrow$ Läge $\vec{r}$

$\rightarrow$ Hastighet $\vec{V}$

$\rightarrow$ Acceleration $\vec{a}$

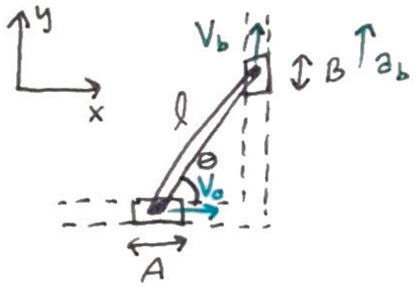


$$\vec{r}(t) = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\vec{v}(t) = V_x\hat{x} + V_y\hat{y} + V_z\hat{z} = \dot{x}\hat{x} + \dot{y}\hat{y} + \dot{z}\hat{z}$$

$$\vec{a}(t) = \dot{V}_x\hat{x} + \dot{V}_y\hat{y} + \dot{V}_z\hat{z} = \ddot{x}\hat{x} + \ddot{y}\hat{y} + \ddot{z}\hat{z}$$

$\boxed{\text{Ex 7.2}}$



Bestäm V_B & a_B som funktion av vinkeln θ

V_0 = konstant

Läget hos A & B:

$$\begin{cases} x_A = -l\cos\theta \\ y_A = 0 \end{cases} \qquad \begin{aligned} x_B &= 0 \\ y_B &= l\sin\theta \end{aligned}$$

$$\vec{r}_A = (-l\cos\theta, 0, 0) \qquad \vec{r}_B = (0, l\sin\theta, 0)$$

Hastigheten hos A & B:

$$\begin{cases} V_{Ax} = V_0 \\ V_{Ay} = 0 \end{cases} \qquad \begin{cases} V_{Bx} = 0 \\ V_{By} = \dot{y}_B = l\cos\theta\cdot\dot{\theta} = l\dot{\theta}\cos\theta \end{cases}$$

$$V_{Ax} = \dot{x}_A = -l\cdot(-\sin\theta)\cdot\dot{\theta} = l\dot{\theta}\sin\theta \Rightarrow V_0 = l\dot{\theta}\sin\theta$$

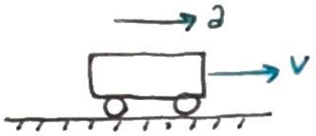
$$\Rightarrow \dot{\theta} = \frac{V_0}{l\sin\theta}$$

$$\Rightarrow V_B = V_{By} = l\dot{\theta}\cos\theta = \frac{\cancel{l}V_0}{\cancel{l}\sin\theta}\cos\theta = V_0\frac{\cos\theta}{\sin\theta} = \frac{V_0}{\tan\theta}$$

$$\Rightarrow a_B = \dot{V}_B = V_0\cdot(\tan\theta)^{-1} = V_0\cdot(-1)(\tan\theta)^{-2}\cdot\frac{1}{\cos^2\theta}\cdot\dot{\theta} =$$

$$= -\frac{v_0\,\dot\theta}{\cos^2\theta\,\tan^2\theta} = -\frac{v_0}{\sin^2\theta}\cdot\frac{v_0}{\ell\sin\theta} = -\frac{v_0^2}{\ell\sin^3\theta}$$

Ex 7.4



$v_0 = 250\ \text{km/h}$

$a = -kv^2 \quad , \quad k = 5\cdot 10^{-4}\ m^{-1}$

Bestäm: $v = v(t) = ?$

$v = v(x) = ?$

Bromssträckan då $v = \frac{v_0}{2}$?

Lösning

Accelerationen:
$$\left.\begin{matrix} a = \ddot{x} \\ a = -kv^2 \end{matrix}\right\} \left(\left.\begin{matrix} \ddot{x} = -kv^2 \\ v = \dot{x} \end{matrix}\right\} \begin{matrix} \ddot{x} = -k\dot{x}^2 \\ \ddot{x} + k\dot{x}^2 = 0 \end{matrix} \right)$$

$$\left.\begin{matrix} a = \dot{v} \\ a = -kv^2 \end{matrix}\right\} \rightarrow \boxed{\dot{v} = -kv^2} \quad (*)$$

VL i $(*)$:

$$\dot{v} = \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = -kv^2 \rightarrow \int\frac{dv}{v^2} = -k\int dt \Rightarrow \frac{v^{-1}}{-1} = -kt + C$$

$$-\frac{1}{v} = -kt + C$$

Begynnelsevillkor (BV):

$$v(t=0) = v_0 \Rightarrow -\frac{1}{v_0} = -k\cdot 0 + C \Rightarrow C = -\frac{1}{v_0}$$

$$\Rightarrow -\frac{1}{v} = -kt - \frac{1}{v_0} \Rightarrow \frac{1}{v} = \frac{kv_0t + 1}{v_0} \Rightarrow \boxed{v = \frac{v_0}{1 + kv_0 t}}$$

Alternativt:

$$\int_{v_0}^{v}\frac{dv}{v^2} = -k\int_0^t dt \Rightarrow \left[-\frac{1}{v}\right]_{v_0}^{v} = -k\,[t]_0^t \Rightarrow -\frac{1}{v} + \frac{1}{v_0} = -k(t-0)$$

$$\Rightarrow -\frac{1}{v} = -kt - \frac{1}{v_0}$$

kedjeregeln:

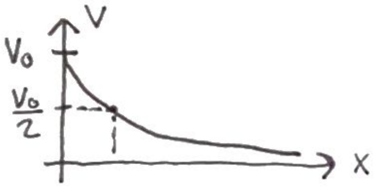
$$\underbrace{\frac{dv}{dt} = \frac{dv}{dx}\cdot\underbrace{\frac{dx}{dt}}_{v}}_{v\frac{dv}{dx}} \quad , \quad \frac{d}{dt}\left[v(x(t))\right] \quad , \quad \frac{d}{dt}\left[\sin\theta\right] = \cos\theta\cdot\dot{\theta}$$

Insättning i (*):

$$v\frac{dv}{dx} = -kv^2 \Rightarrow \int_{v_0}^{v}\frac{dv}{v} = -k\int_0^x dx \Rightarrow \left[\ln v\right]_{v_0}^{v} = -k\left[x\right]_0^x$$

$$\Rightarrow \ln v - \ln v_0 = -kx \Rightarrow \ln\frac{v}{v_0} = -kx \Rightarrow \frac{v}{v_0} = e^{-kx}$$

$$\boxed{v = v_0 e^{-kx}}$$

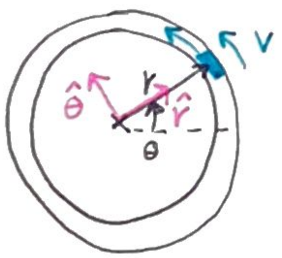


$$v = \frac{v_0}{2} = v_0 e^{-kx} \Rightarrow e^{-kx} = \frac{1}{2} \quad , \quad -kx_1 = \ln\frac{1}{2} = -\ln 2$$

$$\boxed{x_1 = \frac{\ln 2}{k}} \approx 1386 \text{ m}$$

Andra koordinatsystem:

Ex 7.1



Polära ~~koor~~dinater

Cylinderkoordinater

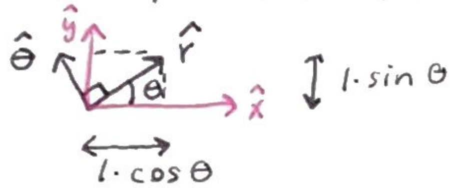
Enhetsvektorer: $\hat{r}, \hat{\theta}, \hat{z}$

$$\vec{r} = r\hat{r}$$

Avs 7.4 - cylinderkoordinater

Lägesvektorn: $\vec{r} = r \cdot \hat{r} + z \cdot \hat{z}$

Hastighetsvektorn: $\vec{v} = \dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\hat{r}} + \dot{z}\hat{z} + z\underbrace{\cancel{\dot{\hat{z}}}}_{=0} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{z}$



$$\hat{r} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y}$$

$$\dot{\hat{r}} = -\dot{\theta}\sin\theta\,\hat{x} + \dot{\theta}\cos\theta\,\hat{y}$$

$$\hat{\theta} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y}$$

$$\Rightarrow \boxed{\dot{\hat{r}} = \dot{\theta}\hat{\theta}}$$

Ex