Mekanikföreläsning 4.1.2. Carlo 22/3-18 Tors

vektoralgebra & kraftmoment

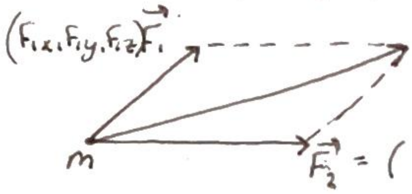
Kapitel 1 Vektoralgebra

I mekaniken: krafter, rörelsemängd

$$\left(\frac{d\vec{P}}{dt} = \vec{F}\right)$$

Vektorer: $\begin{cases} \text{storlek}, & F = |\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2} \quad (\text{belopp}) \\ \text{Riktning}, & \hat{F} = \dfrac{\vec{F}}{|\vec{F}|} = \dfrac{\vec{F}}{F} \quad (\text{enhetsvektor}) \end{cases}$

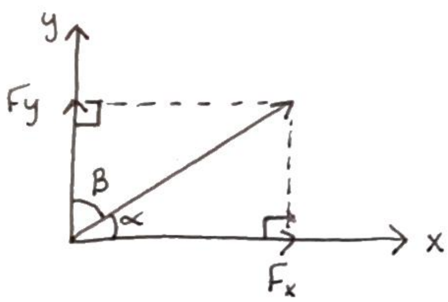
$$\vec{F} = F \cdot \hat{F}$$



$$\vec{F} = \vec{F_1} + \vec{F_2} = (F_{1x} + F_{2x}, F_{1y} + F_{2y}, F_{1z} + F_{2z})$$

$$\vec{F_2} = (F_{2x}, F_{2y}, F_{2z})$$

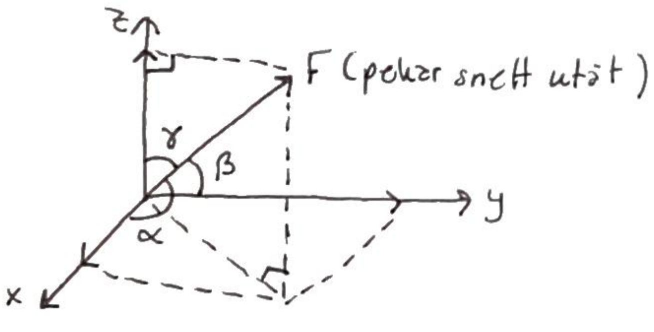
I 2 dimensioner:



$$F_x = F \cdot \cos\alpha$$

$$F_y = F \cdot \cos\beta$$

I 3 dimensioner:



$$F_x = F \cdot \cos\alpha$$

$$F_y = F \cdot \cos\beta$$

$$F_z = F \cdot \cos\gamma$$

Projektion

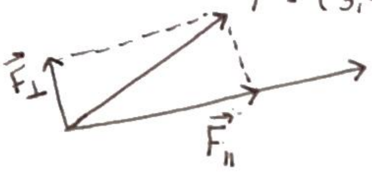
$$F_x = F \cdot \cos\alpha = |\vec{F}| \cdot |\hat{x}| \cdot \cos\alpha = \vec{F} \cdot \hat{x}$$

$$\begin{cases} F_x = \vec{F} \cdot \hat{x} \\ F_y = \vec{F} \cdot \hat{y} \\ F_z = \vec{F} \cdot \hat{z} \end{cases}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Ex 1.2 Sökt: Projektionen av $\vec{F}$ på $\vec{s}$ & projektionen av $\vec{F}$ som är vinkelrät mot $\vec{s}$

$\vec{F} = (3,4,0)$



$\vec{s} = (2,-1,3)$

$$|\vec{F}_\parallel| = \vec{F} \cdot \hat{s} \quad , \quad \hat{s} = \frac{\vec{s}}{|\vec{s}|} = \frac{(2,-1,3)}{2^2+(-1)^2+3^2} = \frac{(2,-1,3)}{\sqrt{14}} = \left(\frac{2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$$

$$\Rightarrow |\vec{F}_\parallel| = \vec{F} \cdot \hat{s} = (3,4,0) \cdot \frac{(2,-1,3)}{\sqrt{14}} = \frac{6-4+0}{\sqrt{14}} = \frac{2}{\sqrt{14}}$$

$$\rightarrow \vec{F}_\parallel = \underbrace{\frac{2}{\sqrt{14}}}_{\text{längd}} \cdot \underbrace{\frac{(2,-1,3)}{\sqrt{14}}}_{\text{Riktning}} = \frac{(2,-1,3)}{7}$$

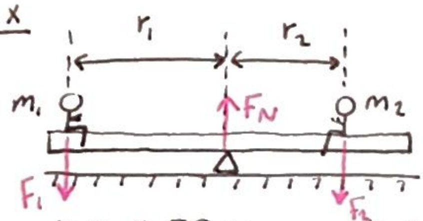
$$\vec{F} = \vec{F}_\parallel + \vec{F}_\perp \quad \rightarrow \quad \vec{F}_\perp = \vec{F} - \vec{F}_\parallel = (3,4,0) - \frac{(2,-1,3)}{7}$$

$$= \left(\frac{19}{7}, \frac{29}{7}, \frac{-3}{7}\right) = \frac{(19,29,-3)}{7}$$

$$|\vec{F} \cdot \hat{s}| = \underbrace{|\vec{F}|}_{\sqrt{3^2+4^2+0^2}=5} \cdot \underbrace{|\hat{s}|}_{1} \cos\alpha = 5\cos\alpha \quad \Rightarrow \cos\alpha = \frac{|\vec{F} \cdot \hat{s}|}{5} = \frac{2/\sqrt{14}}{5} = \frac{2}{5\sqrt{14}}$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{2}{5\sqrt{14}}\right) \approx 84^\circ$$

Ex



$r_1 = 0{,}50\,m$ $\qquad$ $r_2 = 0{,}25\,m$

$m_1 = 30\,kg$ $\qquad$ $m_2 = 40\,kg$

A) ↻

B) ↺

C) Jämvikt

$r_1 \cdot mg \approx 0{,}5\,m \cdot 300\,N = 150\,Nm$ = kraftmoment för 1

$r_2 \cdot mg \approx 0{,}25\,m \cdot 400\,N = 100\,Nm$ = kraftmoment för 2

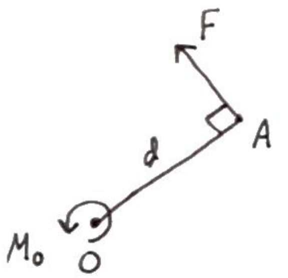
Kraftmoment

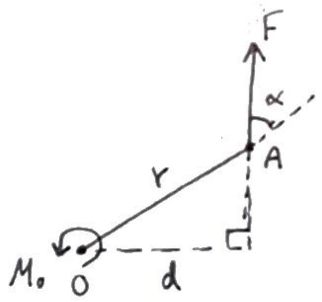
$|\vec{M}| = r \cdot F$

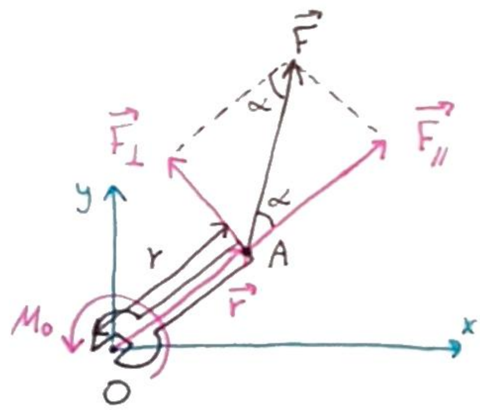
Jämvikt

Kraftjämvikt: $\sum_{k=1}^{n} \vec{F}_k = 0$

Momentjämvikt: $\sum_{k=1}^{n} \vec{M}_k = 0$







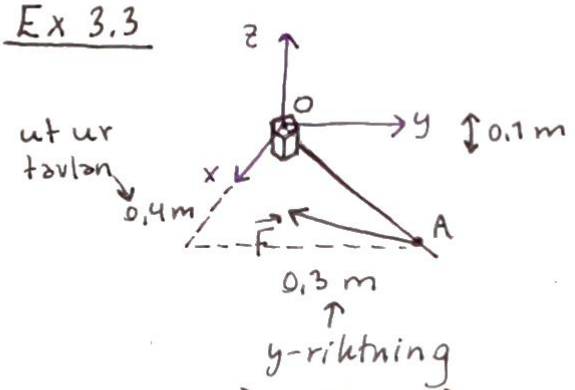
$M_0 = r \cdot F_\perp = r \cdot F \cdot \sin\alpha = |\vec{r}|\,|\vec{F}|\,\sin\alpha = |\vec{r} \times \vec{F}|$, $\vec{r}_{OA} = \vec{r}_A - \vec{r}_O$

Definition $\boxed{\vec{M}_O = \vec{r}_{OA} \times \vec{F}_A}$

lägesvektorn för kraften F:s angreppspunkt $\qquad$ kraften

$$\vec{r} = (r_x, r_y, r_z), \quad \vec{F} = (F_x, F_y, F_z) \quad \Rightarrow \quad \vec{r} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = (r_y F_z - r_z F_y, \; r_z F_x - r_x F_z, \; r_x F_y - r_y F_x)$$

Ex 3.3



$\vec{F} = (-1, -10, 2)$ N

$\vec{M}_o = \,?$

Lösning $\vec{M}_o = \vec{r}_{OA} \times \vec{F} = (0.3 \cdot 2 - (-0.1)(-10), (-0{,}1)(-1) - (0{,}4) \cdot 2, 0.4(-10) - (0{,}3)(-1$

$\vec{r}_{OA} = (0.4, 0.3, -0.1)$ m

$\vec{M}_o = (-0.4, -0.7, -3.7)$ Nm
