1) a) $F = ma = m\frac{dv}{dt}$

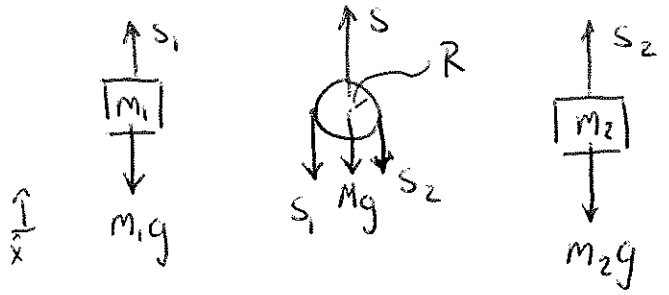
$$\int_{x_a}^{x_b} F \cdot dx = \int_{x_a}^{x_b} m \frac{dv}{dt} \cdot dx = \int_{v_a}^{v_b} m v \, dv =$$

$$= \left[\frac{m v^2}{2}\right]_{v_a}^{v_b} = \frac{m v_b^2}{2} - \frac{m v_a^2}{2}$$

b) $$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} =$$

$$= \underbrace{\vec{v} \times m\vec{v}}_{\vec{v} \parallel \vec{v}} + \underbrace{\vec{r} \times \vec{F}}_{\vec{M}} = \vec{M}$$

2) Frilägg



Ingen rörelse, dvs $a = 0$

a) $S = S_1 + S_2 + Mg = (m_1 + m_2 + M)\, g$

(Anta $m_2 > m_1$)

b)
$$\begin{cases} S_1 - m_1 g = m_1 a & (1) \\ S_2 - m_2 g = -m_2 a & (2) \\ S - S_1 - S_2 - Mg = 0 & (3) \\ R \cdot S_2 - R \cdot S_1 = I\alpha = \frac{1}{2} M R^2 \alpha = \frac{1}{2} M R a & (4) \end{cases}$$

$$\frac{(4)}{R} \Rightarrow S_2 - S_1 \overset{(1)+(2)}{=} -m_2 a + m_2 g - m_1 a - m_1 g = \frac{1}{2} M a$$

$$\Rightarrow a = \frac{m_2 g - m_1 g}{m_1 + m_2 + \frac{1}{2} M}$$

$$(3) \Rightarrow S = S_1 + S_2 + Mg = m_1 \frac{m_2 g - m_1 g}{m_1 + m_2 + \frac{1}{2} M} + m_1 g +$$

$$+ m_2 g - m_2 \frac{m_2 g - m_1 g}{m_1 + m_2 + \frac{1}{2} M} + Mg =$$

$$= m_1 g + m_2 g + Mg + (m_1 - m_2) \frac{(m_2 g - m_1 g)}{m_1 + m_2 + \frac{1}{2} M} =$$

$$= m_1 g + m_2 g + Mg - \frac{(m_1 - m_2)^2 g}{m_1 + m_2 + \frac{1}{2} M}$$

3)

$$\ddot{\theta} = \alpha$$

$$\dot{\theta} = \alpha t + c_1$$

$$\theta = \frac{\alpha t^2}{2} + c_1 t + c_2$$

Cyklisten startar i vila

$$\Rightarrow \quad c_1 = c_2 = 0$$

Tiden för ett varv

$$2\pi = \frac{\alpha t^2}{2}$$

$$t = \sqrt{\frac{4\pi}{\alpha}}$$

Acceleration i polära koordinater

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\,\hat{\theta}$$

$R = \text{konst} \Rightarrow \ddot{r} = \dot{r} = 0$

$$\Rightarrow \quad \vec{a} = -R\alpha^2 t^2 \hat{r} + R\alpha\hat{\theta} =$$

$$= -R\alpha^2 \frac{4\pi}{\alpha}\hat{r} + R\alpha\hat{\theta} =$$

$$= -R\cdot 4\pi\alpha\,\hat{r} + R\alpha\hat{\theta}$$

Hast. i polära koordinater

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = R\cdot\alpha t\,\hat{\theta} =$$

$$= R\alpha\sqrt{\frac{4\pi}{\alpha}}\,\hat{\theta}$$

4) Energins bevarande. (först löser vi för cylindern)

$$E_i = mgL \cdot \sin\alpha$$

$$E_e = \frac{mv^2}{2} + \frac{I\omega^2}{2}$$

translationsenergi rotationsenergi

$$mgL \cdot \sin\alpha = \frac{mv^2}{2} + \frac{1}{4} mr^2\omega^2 =$$

$$= \frac{mv^2}{2} + \frac{1}{4} mv^2 =$$

$$= \frac{3mv^2}{4}$$

$$v_{cylinder} = \sqrt{\frac{4gL\sin\alpha}{3}} = \sqrt{\frac{28gL\cdot\sin\alpha}{21}}$$

För en stav blir lösningen istället.

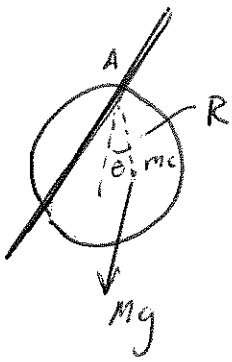
$$MgL\sin\alpha = \frac{Mv^2}{2} + \frac{2}{2\cdot 5} mR^2\omega^2 =$$

$$= \frac{7Mv^2}{10}$$

$$v_{stav} = \sqrt{\frac{10gL\cdot\sin\alpha}{7}} = \sqrt{\frac{30gL\cdot\sin\alpha}{21}}$$

$$v_{stav} > v_{cylinder}$$

5/ Svängningen sker i kontakten mellan ringen och stången.



Steiners sats ger

$$I = I_{mc} + MR^2 = MR^2 + MR^2$$

$$= 2MR^2$$

För den fysikaliska pendeln gäller

$$M = I\alpha$$

$$-R \cdot Mg \cdot \sin\theta = 2MR^2\ddot{\theta}$$

$$\Rightarrow \quad \ddot{\theta} + \frac{g}{2R}\sin\theta = 0$$

Små vinklar $\sin\theta \approx \theta$

$$\Rightarrow \quad \ddot{\theta} + \frac{g}{2R}\theta = 0 \qquad \text{Harm. diff. ekv.}$$

$$\theta(t) = A\sin\omega t + B\cos\omega t \quad ; \quad \omega = \sqrt{\frac{g}{2R}}$$

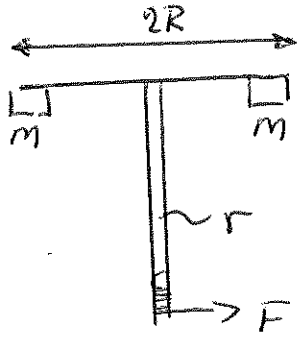
Begynnelsevilkor $\theta(t=0) = 0$; $R\dot{\theta}(t=0) = v$

$$\theta(t=0) = 0 \Rightarrow B = 0$$

$$R\dot{\theta}(t=0) = v \Rightarrow A = \frac{v}{\omega R}$$

$$\Rightarrow \quad \theta(t) = \frac{v}{\omega R}\sin\omega t$$

6)



$$\vec{M} = I\alpha$$

$$r \cdot F = 2mR^2\alpha$$

$$\Rightarrow \quad \alpha = \frac{r \cdot F}{2mR^2} \quad \text{konst.}$$

$$\Theta(t) = \frac{\alpha t^2}{2} = \frac{r \cdot F}{4mR^2} \cdot t^2$$

Alt. Energiprincipen

$$F \cdot s' = F \cdot \frac{r}{R} s = \frac{I\omega^2}{2} = \frac{2mR^2\omega^2}{2} = \frac{2mv^2}{2}$$

sträckan som F verkar

sträckan som massorna har

$$\Rightarrow \quad v^2 = \frac{F \cdot r \cdot s}{mR} = 2a \cdot s$$

konst acc.

$$\Rightarrow \quad a = \frac{F \cdot r}{2mR} = R \cdot \alpha$$

$$\Rightarrow \quad \alpha = \frac{F \cdot r}{2mR^2}$$

$$\Rightarrow \quad \Theta(t) = \frac{\alpha t^2}{2} = \frac{F \cdot r}{4mR^2} t^2$$