25/4-18 Föreläsning 9

1) PID controllers in frequency domain
2) Advanced PID design

socrativ:

stability margin is a <u>length</u>

$$\min_s \left| L(s) - (-1) \right| = \min_s \left| L(s)+1 \right| = \max_s \left| S(s) \right|$$

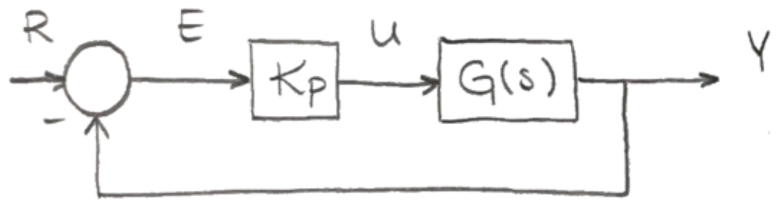
$$\frac{1}{|L(s)+1|} = |S(s)| \leftarrow$$ sensitivity fcn.
(sensitivity margin does not exist).

3) Non-minimum phase models

1) PID controllers in frequency domain

a) $C(i\omega) = K_p \quad (0 < K_p < \infty)$

proportional controller



$u = C(i\omega)E = K_p E$

K_p has no denpendency on $\omega \in [0+\infty)$

$\Rightarrow$ M $20\log|K_p| = K_{p[dB]} =$ constant

$\Rightarrow$ P $\varphi(\omega) = 0$ no phase shift

b) PD $C(i\omega) = K_p(1 + T_d i\omega)$ $K_p < 1$

Bode plot?

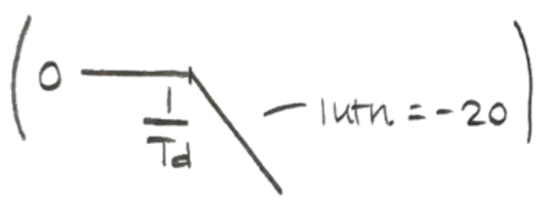
magnitude $20\log|C(i\omega)|$

phase shift $\varphi(\omega)$

Magnitude

$$\underset{\text{constant}}{K_p} \cdot \underbrace{(1 + T_d i\omega)}_{= \left(\frac{1}{1+T_d i\omega}\right)^{-1}}$$

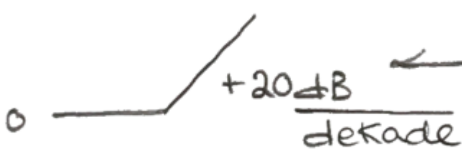
(When $i\omega = \frac{1}{T_d}$ drop to $-\infty$ with "lutning" -20.)

$\rightarrow$ (

$)^{-1}$ first order lag.

(plot inversion is mirroring onto ω-axis)

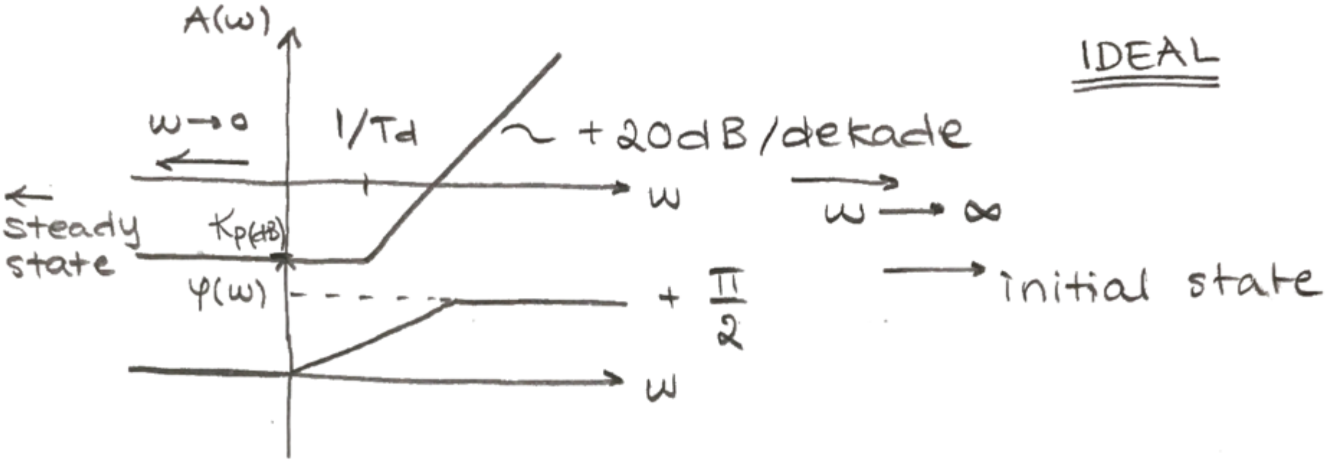
$(1 + T_d i\omega) \cdot K_p = C(i\omega)$



mirror

$K_p < 1$

$K_p\,[dB] < 0$



⊕ Phase added, advancing up to $+\frac{\pi}{2}$ (extra phase)

⊖ It amplifies high frequency signals ($E(i\omega)$ $\omega\uparrow$) (Bad with noise!)

To solve the problem of ∞ amplification $\omega \to \infty$

Realistic PD

LEAD compensator – version of PD-controller

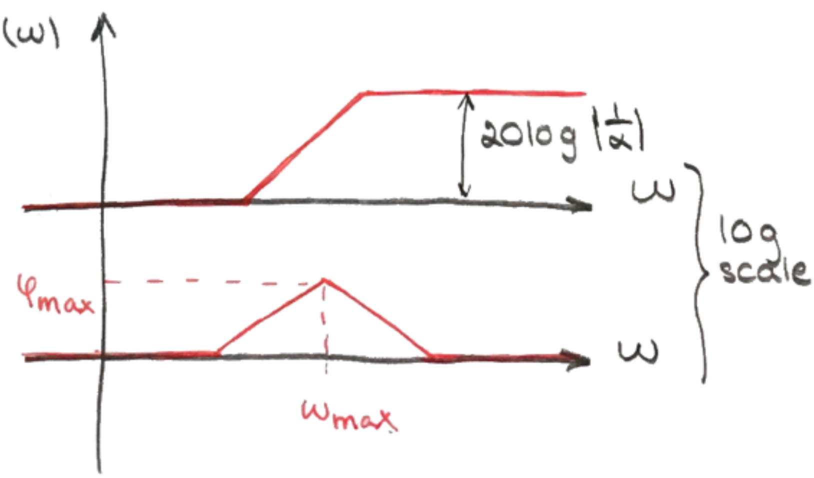
$$C(i\omega) = \frac{T_l i\omega + 1}{\alpha T_l i\omega + 1}$$; T_l – lead time constant, $\alpha < 1$

Solving our problem

$$\lim_{\omega \to \infty} C(i\omega) = \frac{1}{\alpha} \quad \text{(finite value)}$$

$$\lim_{\omega \to 0} C(i\omega) = 1$$

Lead compensator:



How to tune?

phase margin

Step 1: Main idea: $\varphi_m + \varphi_{max}$

Know $\varphi_m, \omega_c \xrightarrow[\text{find}]{} \varphi_{max} = \varphi_m - (\pi + \varphi(\omega_c))$

$$\underbrace{\sin(\varphi_{max}) = \frac{1-\alpha}{1+\alpha}}$$

$$\Rightarrow \alpha = \frac{1-\sin(\varphi_{max})}{1+\sin(\varphi_{max})}$$

$$\omega_{max} = \frac{1}{T_l\sqrt{\alpha}} \quad ; \quad T_l = \frac{1}{\sqrt{\alpha}\cdot\omega_c}$$

• Key parameters

c) PI $\quad C(i\omega) = K_p\left(1 + \frac{1}{T_i \cdot i\omega}\right)$

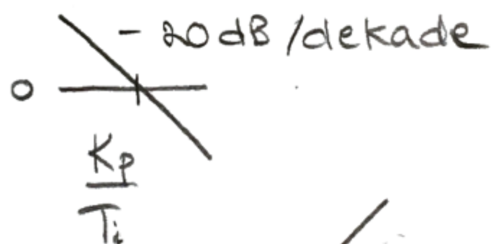
(IDEAL)

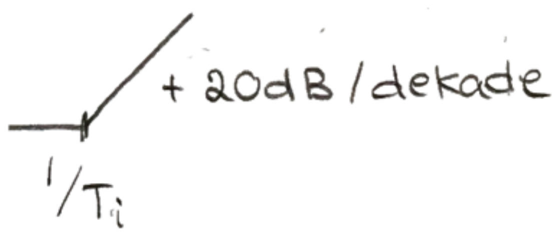
T_i - integrator time constant

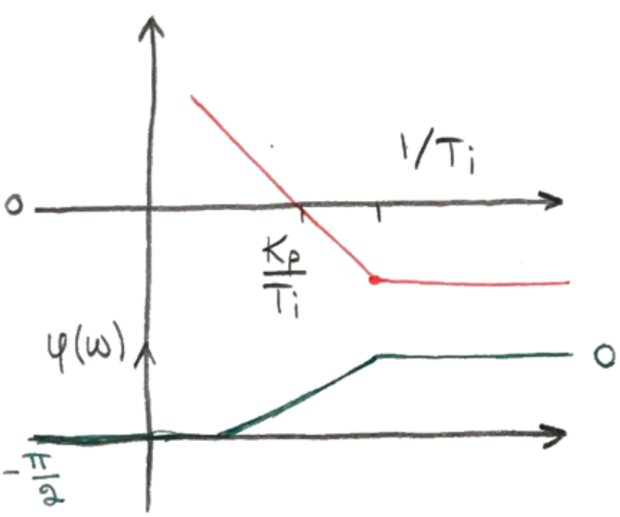
$C(i\omega) = \frac{K_p}{T_i \cdot i\omega}(T_i \cdot i\omega + 1)$, suppose $K_p < 1$

$A(\omega) = 20\log|C(i\omega)|$

integrator?







← frequency plot

$$u(i\omega) = C(i\omega)E(i\omega)$$
(C is shaping E into u)

HW: $\lim_{\omega \to \infty} C(i\omega) = ?$

$$\lim_{\omega \to 0} C(i\omega) \longrightarrow \infty$$

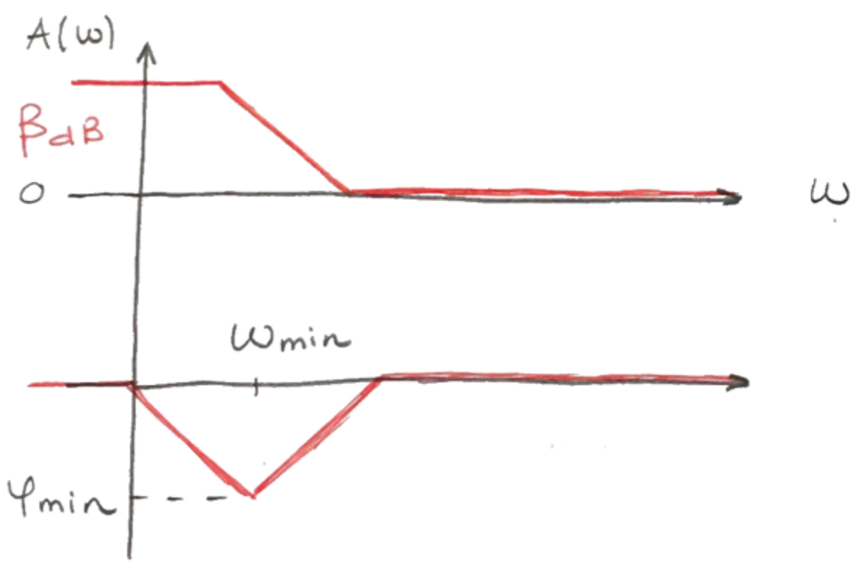
⊕ We anihilate error
⊖ $\omega \to 0$ (steady state)
$C(i0) \uparrow \infty$
$\Rightarrow \omega \to 0$ unstable

Realistic PI controller (LAG compensator)

$$C(i\omega) = \beta \frac{T_l i\omega + 1}{\beta T_l i\omega + 1}$$

T_l - log time constant $\beta > 1$

$$\lim_{\omega \to 0} C(i\omega) = \lim_{\omega \to 0} \frac{\beta (T_l i\omega + 1)}{\beta T_l i\omega + 1} = \beta$$



$\beta \uparrow$ but finite

2) Advanced PID controllers

A) Cascade controller design

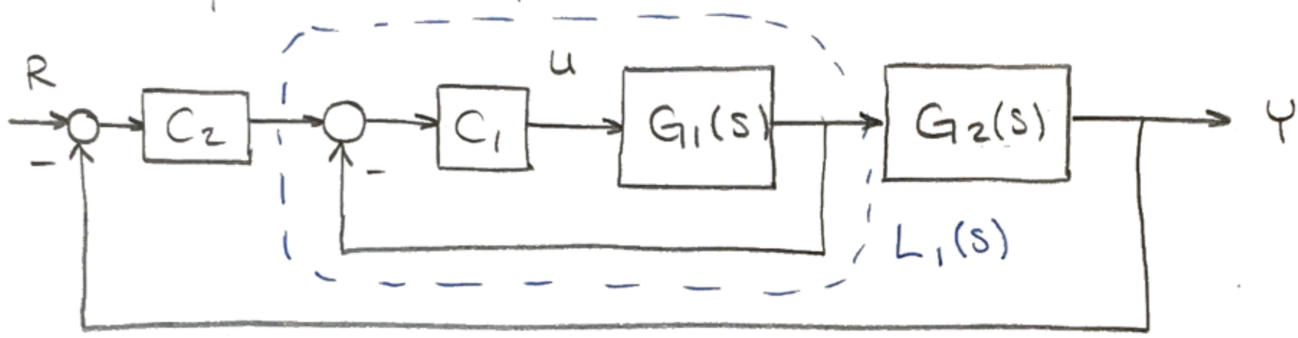
P: There is a slow and a fast dynamic components

S: Let's control them "separately"

Loop in a loop controller solution.

$$G(s) = \underbrace{G_2(s)}_{\text{slow}} \underbrace{G_1(s)}_{\text{fast}}$$

Assumption: $G_1(s) \cdot u(s)$ - measurable



Loop-transfer

$$L_1(s) = \frac{C_1 C_2}{1 + C_1 C_2}, \quad L(s) = C_2(s) L_1(s) G_2(s)$$

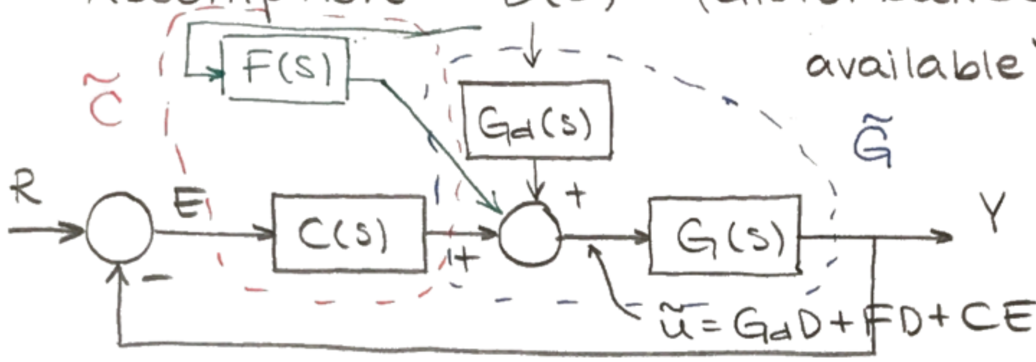
$$T(s) = \frac{C_2(s) L_1(s) G_2(s)}{1 + C_2(s) L_1(s) G_2(s)}$$

B) Disturbance feedforward

P: How to change the controller to use $D(s)$?

S: If $D(s)$ measurable let's compensate for it.

Assumption $D(s)$ (disturbance available)



$D(s)$ - disturbance, measured

$G_d(s)$ - effect of $D(s)$ on the input

$\tilde{G}$ - it has a disturbance at (the plant) input.

e.g. Temperature control of the lecture hall.
Enviroment (T_e) influence the room temp.

$$\tilde{C}(s) = [C(s) \ F(s)]$$

$$u(s) = \tilde{C}(s) \begin{bmatrix} E(s) \\ D(s) \end{bmatrix} = CE + FD$$

$\left.\begin{matrix} F(s) \\ C(s) \end{matrix}\right\}$ Two degrees of freedom

$$E = R - Y = R - G(CE + FD + G_d D$$

$$E(1 + GC) = R - (F + G_d)GD$$

$$E = \underbrace{\frac{1}{1+GC}}_{\text{closed loop}} R - \underbrace{\frac{1}{1+GC}(F + G_d)GD}_{\text{effekt of } D}$$

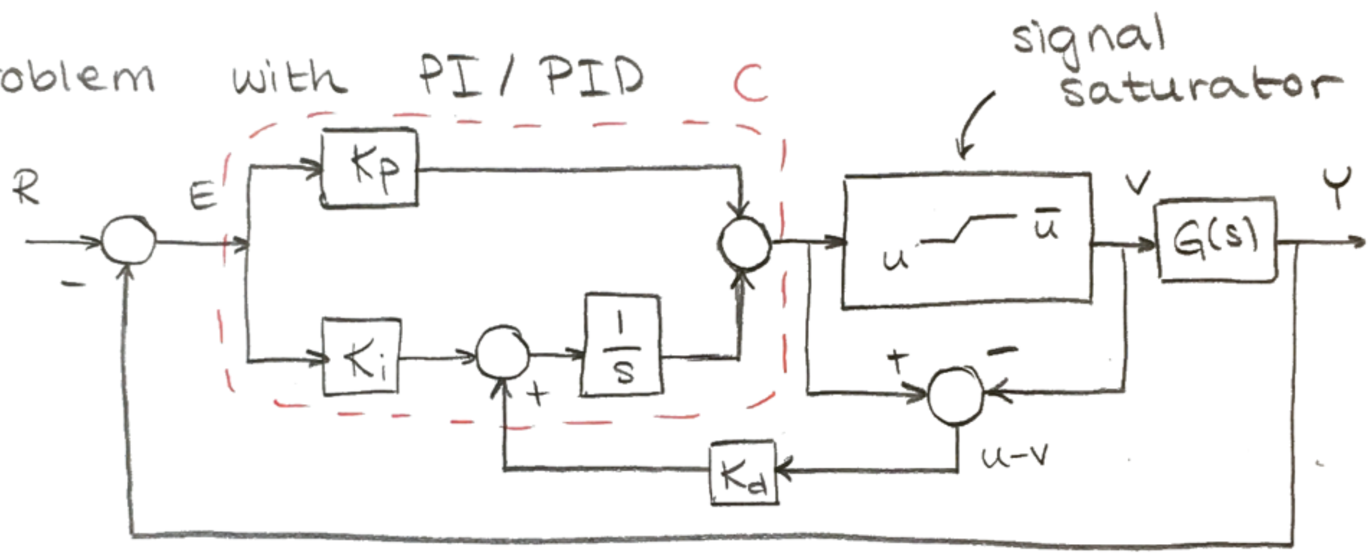
$-F = G_d$ ← this choise will annulate the effekt of D.

c) Anti-windup

P: $\underline{u} < u(t) < \bar{u}$ is bounded

(e.g. Balanduino robot DC motor finite torque)
Performance degradation is the main problem.
S: Try to involve the level of being saturated.

Problem with PI / PID



Informed the PI controller about saturation
Main: error cumulation is stopped