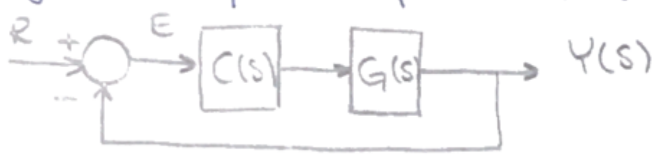
23/4-18 Föreläsning 8

Design output feedback controllers



1. Struktured $C(s)$ ($\leftarrow$ PID)

$$u(s) = \left(K_p + \frac{K_i}{s} + K_d s\right) E(s)$$

2. Methods – heuristics (ZN)
   – guaranteed stab.

Nyquist stability theorem:

to guarantee closed-loop stab. ($T(s)$) by open-loop methods ($L(s)$)

$L(s) = G(s)C(s)$.

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1) frequency function
2) Bode stability conditions
3) PID controllers

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1) Frequency function

Aim: guaranteed stability condition for closed-loop; a "new" idea (Bode)

Transfer fcn.

$$G(s) = \frac{Y(s)}{U(s)}$$

$$U(s) = \mathcal{L}\{\underbrace{\delta(t)}_{\text{dirac}}, \underbrace{1(t)}_{\text{step}}, \underbrace{t}_{\text{ramp}}\}$$

$u(s)$ can be arbitrary.

Can we define a "metric", tranfer that ralates more to frequency domain?

e.g. $u(t) = \sin(t)$

Idea: to check the controller closed-loop in "non-time" domain, frequency.

Def. frequency function

$$G(s)\Big|_{s=i\omega} = G(i\omega)$$

formally obtained from $G(s)$ $\omega \in [0, +\infty)$, physically meaningful.

$$u(i\omega) \longrightarrow \boxed{G(i\omega)} \longrightarrow Y(i\omega)$$

$u(t) = 1 \cdot \sin(\omega t + 0)$

We reduce the set of input signals to unity magnitud and ω dependent phase. Chirp with ω and check magnitude / phase shift.

What is $y(t)$ with $G(i\omega)$?

Suppose: $u(t) = \sin(\omega t)$

$$y(t) = \int_0^\infty g(\tau) u(t-\tau) d\tau \quad ; \quad u(t) = \operatorname{Im}(\cos(\omega t) + i \sin(\omega t)) = \operatorname{Im}(e^{i\omega t})$$

$$y(t) = \int_0^\infty g(\tau) \sin(\omega(t-\tau)) d\tau =$$

$$= \operatorname{Im}\left(\int_0^\infty g(\tau) e^{i\omega(t-\tau)} d\tau\right) = \operatorname{Im}\left(e^{i\omega t} \overbrace{\int_0^\infty g(\tau) e^{-i\omega\tau} d\tau}^{G(i\omega)}\right) =$$

$$= \operatorname{Im}\left(e^{i\omega t} G(i\omega)\right) = \operatorname{Im}\left(|G(i\omega)| e^{i\varphi(\omega)} \cdot e^{i\omega t}\right)$$

with $\varphi(\omega)$ being the phase shift with $G(i\omega)$

$$y(t) = |G(i\omega)| \cdot \mathrm{Im}\left(e^{i\varphi(\omega) + \omega t}\right) =$$
$$= |G(i\omega)| \sin(\omega t + \varphi(\omega))$$

$|G(i\omega)| \geq 1$ amplification of input magnitude

< 1 attenuation

$\varphi(\omega)$ phase shift $-$ delays, $+$ advances

There will be a change in the magnitude and phase, depending on ω and system model.

How to check/plot frequency function.

A) Nyquist - polar coordinates; plot of complex vectors.

B) Bode plot - co-axial plot of $\varphi(\omega)$ and $|G(i\omega)|$ separately.

E.g. $G(s) = 1/(\tau s + 1)$

i) FF with $\tau = 1$

ii) $y(t) = ?$ $\tau = 1s$, $\omega = 1\ rad/s$

iii) Nyquist, Bode plots

(i–ii) $G(s)\Big|_{s=i\omega} = \dfrac{1}{i\omega + 1}$, $G(i) = \dfrac{1}{1+i}$ $\omega = 1$

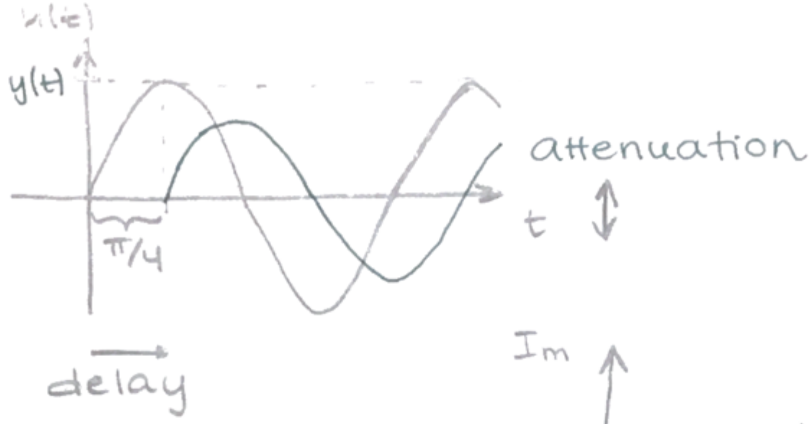
$|G(i)| = ?$ $\varphi(\omega) = ?$

$G(i) = 1/(1+i) = \frac{1}{2} - \frac{i}{2}$

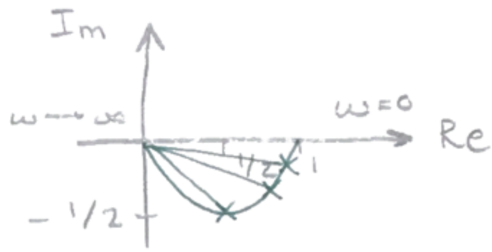
$|G(i)| = \sqrt{(1/2)^2 + (1/2)^2} = \sqrt{2}/2 < 1$

$\varphi(\omega) = \varphi(1) = \tan^{-1}(-0{,}5/0{,}5) = -\frac{\pi}{4}$

$$y(t) = -\frac{\sqrt{2}}{2} \sin\left(t + \left(-\frac{\pi}{4}\right)\right)^{-1}$$



(iii) $G(i\omega) = \dfrac{1}{1+\omega}$



$G(0) = 1$

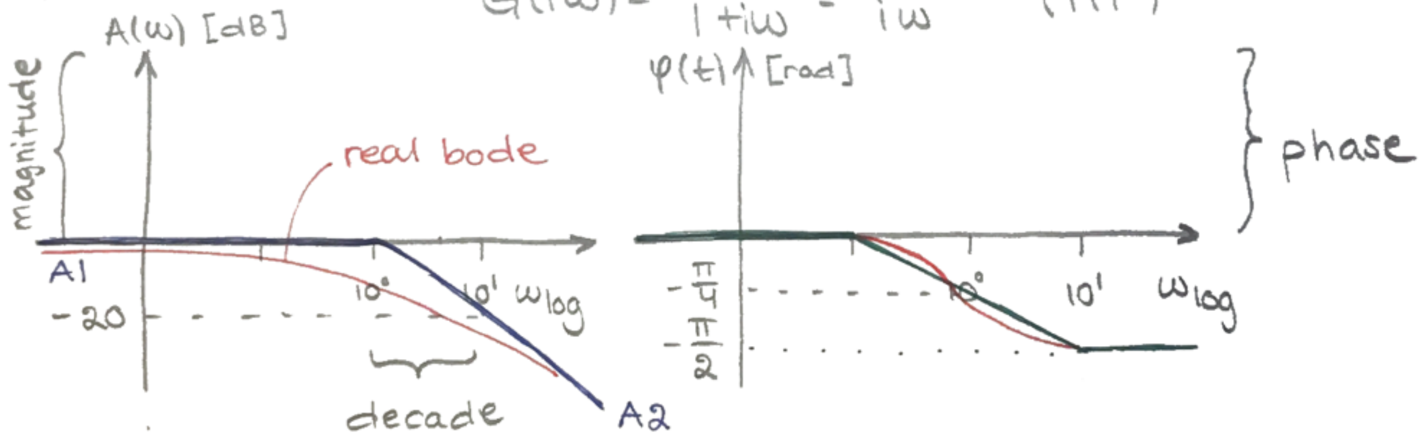
$\varphi(\omega) \in [0 - \frac{\pi}{2})$

$G(\omega \to \infty) \to 0 \qquad |G(i\omega)| \in [1\ 0)$

Bode diagram (A - asymptot)

A 1) $\omega \ll 1$ $\qquad G(i\omega) = \dfrac{1}{1+i\omega} \approx 1$ (low frequency asymptot) (LF)

A 2) $\omega \gg 1$ $\qquad G(i\omega) = \dfrac{1}{1+i\omega} = \dfrac{1}{i\omega}$ (HF)



$A(\omega) = 20 \cdot \log_{10} |G(i\omega)| \quad [dB]$

$LF \overset{[dB]}{\approx} -20 \log_{10} |i\omega|$

Remark for A) Nyquist FF plot :

(Set of basic FF elements)

$$L_i(i\omega) = \left\{ \frac{A_i}{1+i\omega},\ A_d i\omega,\ \frac{A_i}{i\omega} \right\}$$

$$L(i\omega) = \sum_{i=1}^{l} L_i(i\omega)$$

sum of basic elements for plotting

B) Bode diagram (due to logarithmic scale)

$$L(i\omega) = \prod_{i=1}^{m} L_i(i\omega)$$

product of baseline components. For plotting of FF Nyquist / Bode is decomposed into a sum / product of basic element.

2) Bode stability condition

Main idea: use $L(i\omega)$ (FF of the loop and design stable controllers).

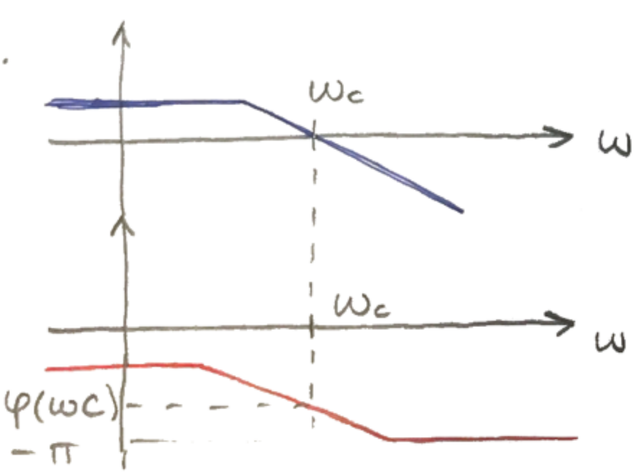
Theorem: (Bode compensation)

The closed-loop LTI model is input-output stable iff $L(i\omega)$ (open-loop frequency fcn, $p_{OL}=0$) has a strictly positive phase margin $\varphi_m > 0$. (asymptotic stability).

In other words: $|L(i\omega_c)| = 1$, ω_c - cut frequency (cutting, crossover)

$$\varphi_m \stackrel{\Delta}{=} \varphi(\omega_c) + \pi > 0$$

ex.



$\varphi_m > 0 \quad \varphi(\omega_c) > \pi$

Algoritm (Bode compensation)
(closed-loop with some extra margin)

1. select $\varphi_m \approx 30-60^\circ$
φ_m - is a metric how much phase surphus margin, we add.

2. $L(i\omega)$ - open-loop frequency function
$= G(i\omega) \cdot C(i\omega)$

$C(i\omega) = \left(K_p + K_d i\omega + \frac{K_i}{i\omega}\right)$, K_p, K_i, K_d are yet unknown

Fix values; $K = \{K_d, K_i, K_p\}$ (initial guess) | you create a controlled loop (may be unstable)

3. Guarantee stability + φ_m

K from step 2 may have wrong φ_m

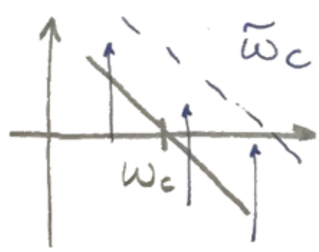
$\varphi(\omega_c) + \pi$

stable ← positive

unstable ← negative

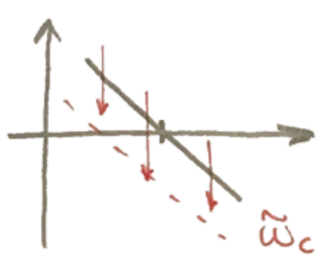
$\varphi(\omega_c) + \pi < 0$

K selected in step 2 is not large enough
⇒ Increase K(↑), take one of k's parameter.



$\varphi(\tilde{\omega}_c) + \pi = \varphi_m$

$\varphi(\omega_c) + \pi > \varphi_m$
K has to be lowered.



3) PID controllers

How to tune PID to guarantee Bode criteria?

Frequency behaviour of PID controllers.

A) $C(i\omega) = K_p$ - proportionel

- static control
- static magnitude behaviour

$$A_p = A(\omega)\Big|_{K_p} = 20 \cdot \log|K_p| \qquad \varphi(\omega) = 0$$

$\Rightarrow$ A_p is ω-independent, no phase shift.

B) PD - proportionel - dift

$C(i\omega) = K_p \cdot (1 + T_d i\omega)$

constant