18/4-18 Föreläsning 7

- Nyquist stability theory relation between open and closed-loop.
- Argument variation principle
- Stability by Nyquist

$\dot{x}(t) = Ax(t)$, x_0 (Socrative)

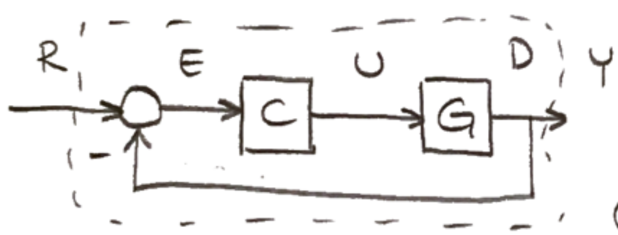
$$x(t) = e^{At} \cdot x_0 \Big|_{u=0}$$

state transition matrix

Avoid mixing it with similarity state-transform.

$\tilde{x}(t) = Tx(t)$

(socrative)



$G_c = T$

$$T(s) = \frac{Y(s)}{R(s)} = \frac{L(s)}{1+L(s)}$$

+

$$S(s) = \frac{Y(s)}{D(s)} = \frac{1}{L(s)+1}$$

$1 - T = S$

PID (socrative)

$$C(s) = K_p + \frac{K_i}{s} + K_d s$$

P I D

dynamics structured

not state! It is output

$$U(s) = C(s)E(s) = C(s)(\underbrace{R(s) - Y(s)}_{\text{output}})$$

output feedback

there is no state.

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1. Nyquist stability theory

Aim: With the loop transfer function $L(s) = G(s)C(s)$ decide whether the closed-loop is stable or not,

$T(s) = G_c(s)$

1) Relation $T(s) \longleftrightarrow L(s)$

2) How to do?

Note, nyquist stat. criteria fits into the controller tuning methods (L6)

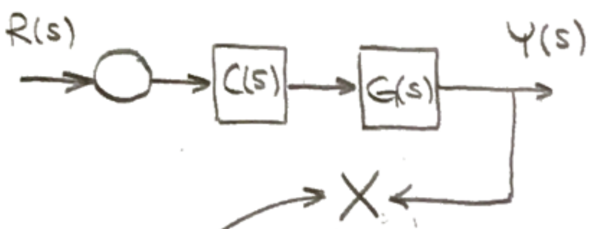
1) Heuristics

2) Guaranteed stability

2. Relation between open and closed-loop

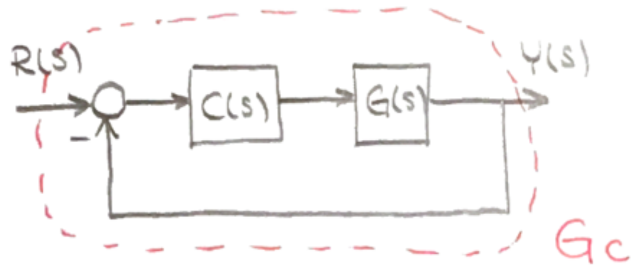
Blockdiagrams

OPEN



opening the loop.

CLOSED



T

$$T(s) = G_c(s) = \frac{L(s)}{1+L(s)} = \frac{G(s)C(s)}{1+G(s)C(s)} = \frac{b(s)}{a(s)+b(s)}$$

$$L(s) = \frac{b(s)}{a(s)}$$

$$f(s) = L(s) + 1 = \frac{b(s)+a(s)}{a(s)}$$

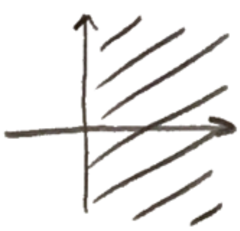
return ratio.

Pole polynomial for $T(s)$ is identical to the zero polynomial of $f(s)$

$f(s) \downarrow Z_{OL}$ (zeros of open-loop)

$T(s) \downarrow P_{CL}$ (poles of closed-loop)

We aim at finding Z_{OL} over RHP



Need for searching over RHP:
Technique to seek is by means of Cauchy integral criteria.

Argument variation principle

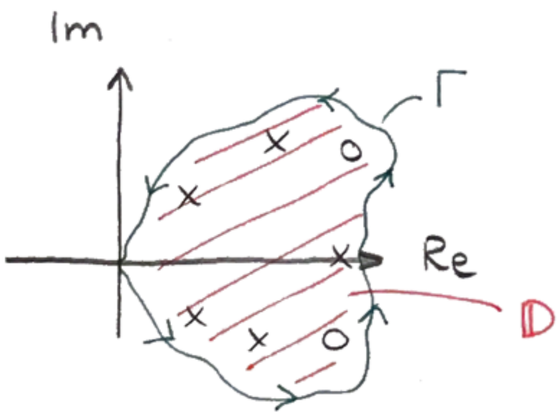
Theorem Let $f(s)$ be holomorphic on $\mathbb{D}$ (domain) closed by a (piecewise smooth curve) Γ (no poles, no zeros on Γ) except a finite number of zeros and poles. Then,

$$\boxed{z - p = \omega}$$

where z/p is the total # of zeros/poles of $f(s)$ over $\mathbb{D}$.
Including multiplicities (repeated poles/zeros at the same location)
And ω - is a winding/encirclement # around orgin of $\mathbb{C}$.
We will select $\mathbb{D}$ to be the RHP.

e.g.

domain for $f(s)$

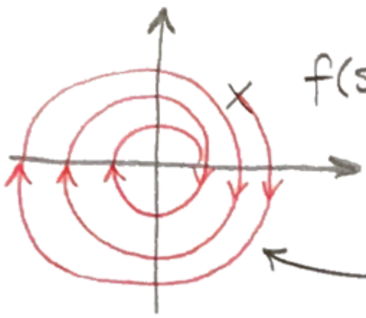


o - zeros
x - poles

Γ tipping direction CCW positive $\circlearrowleft +$

$$z - p = 2 - 5 = \underbrace{-3}_{\text{spinning/winding around the orgin}}$$

$f(s) \quad s \in \mathbb{D}$

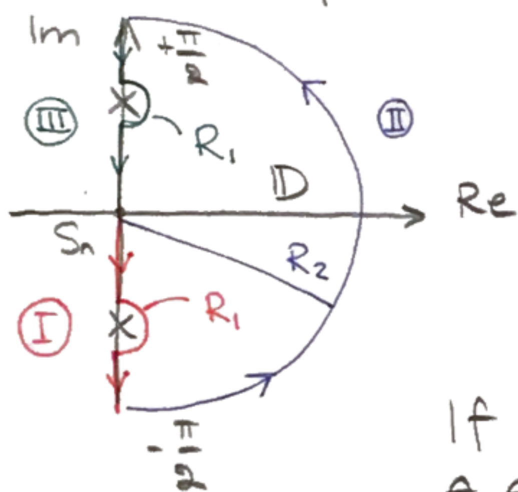


$f(s_i) = f(s_f)$

ty -3 är negativ byter man håll på varven.

Let's map RHP ← $\mathbb{D}$

How to define $\mathbb{D}$?



X - poles

<u>Nyquist contour</u>

<u>step I:</u>

$s = -i \cdot \omega$

If there is a pole $R_1 e^{i\theta}$ $\quad R_1 \to 0$

$\theta \in [+\frac{\pi}{2}, -\frac{\pi}{2}]$

<u>Step II:</u>

$s = R_2 e^{i\theta} \qquad \theta \in [-\frac{\pi}{2}, +\frac{\pi}{2}], \quad R_2 \longrightarrow \infty$

<u>Step III:</u>

$s = +i\omega \qquad \omega (\infty, 0]$

Γ + I II III

Note,

$$Z_{OL} - P_{OL} = W_{OL}$$

$$P_{CL} = Z_{OL}$$

$$P_{CL} - P_{OL} = W_{OL}$$

$$P_{CL} = \underbrace{W_{OL} + P_{OL}}_{\text{open-loop} \Rightarrow \text{closed-loop}}$$

3. Stability by Nyquist

Theorem The closed-loop is asymptotically input-output stable (no $P_{CL} = 0$ at RHP)

$$P_{CL} = W_{OL} + P_{OL}$$

where P_{CL} is the number of closed-loop poles.

P_{OL} is # of open-loop poles.

W_{OL} is the encirclement of $f(s)$ around $\mathbb{C}$

Nyquist simplified further the stability theorem as $P_{OL} = 0$ (no unstable open-loop poles), $f(s) = L(s) + 1$

encircleling origin $\mathbb{C}$

$\Leftrightarrow$

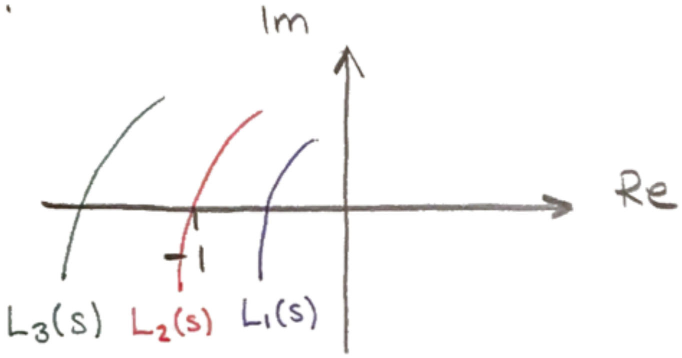
w/ $L(s)$ encircle -1.

Theorem (Simplified Nyquist theorem)

The closed-loop is asymptot stable (input-output) iff $p_{CL} = 0$ and if $p_{OL} = 0$ then $p_{CL} = \omega_{OL} = 0$

ω_{OL} is the encirclement around (real) -1 point.

e.g.



$L_1(s) \Rightarrow \omega_{OL} = 0$ if $p_{OL} = 0$

$\Rightarrow p_{CL} = 0 \Rightarrow$ stable

$L_2(s) \Rightarrow$ marginally stable (Lyapunov stable)

$L_3(s) \Rightarrow \omega_{OL} \neq 0 \Rightarrow p_{CL} \neq 0$ unstable

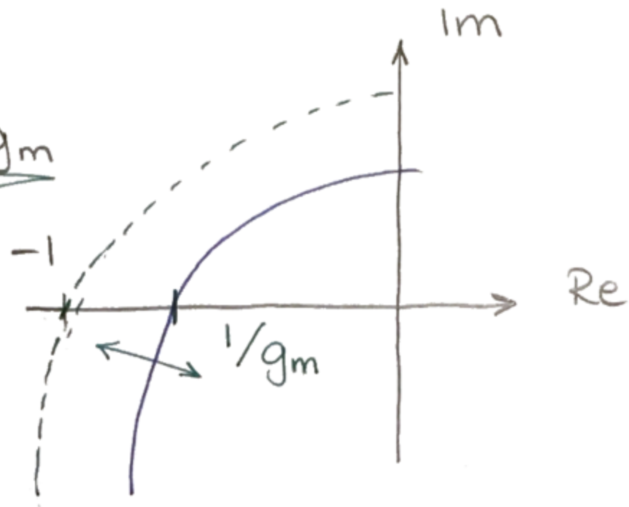
We looked at $L(s)$ and concluded stability for $T(s)$ (closed-loop)

What if a closed-loop is stable, how much surplus / margin we have before getting unstable

1) Gain margin

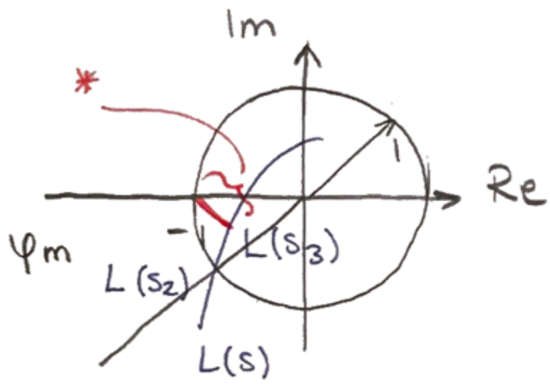
$0 < g_m < \infty$

$\underline{g_m} L(s_1) = |L(s_1)| e^{i\varphi(s)}$ $\underline{g_m}$



2) Phase margin

$L(s_2) = |L(s_2)| e^{i(\varphi(s) + \varphi_m)}$ ← phase margin



3) Stability margin

(*) stability margin, s_3

$$\min_s |L(s) - (-1)| = \min_s |\underbrace{L(s) + 1}_{f(s)}|$$

$\Longleftrightarrow$

$$\max_s \left| \frac{1}{L(s)+1} \right| = \max_s |S(s)|$$

↑ sensitivity function