26/3-18 Föreläsning

1) Basline models (time domain)
- pure integration
- first order lag
- second order lag

2) Interconnections of system models
3) Closed-loop stability

Ex. $G(s) = \frac{1}{s^2+2s+4}$, stable?

a) find poles of $\frac{1}{a(s)}$, $a(s)\Big|_{p=?} = 0$

$\Rightarrow p_1 = -1 \pm \sqrt{3}\, i$

real part negative $\Rightarrow$ stable

b) do it with Routh matrix, use it if degree of a(s) is large (>2)

Idea: check the coeffs. of $a(s) = s^n + a_1 s^{n-1} + \ldots + a_n$
$\{a_0, a_1, \ldots, a_n\}$

even nr. coeffs. $\{a_0, a_2, \ldots\}$
odd nr. coeffs. $\{a_1, a_3, \ldots\}$

$as = \underset{a_0}{\underline{1}} s^2 + \underset{a_1}{\underline{2}} s + \underset{a_2}{\underline{4}} s^0$

Use even/odd to create rouths in routh matrix.

s^2	$1 = a_0$	$4 = a_2$	0	$\leftarrow$ even #
s^1	$2 = a_1$	$0 = a_4$	0	$\leftarrow$ odd #
s^0	$b_1 = 4$ $c_1 = 0$	$b_2 = 0$ $c_2 = 0$		

$$b_1 = \frac{2 \cdot 4 - 0}{2} = 4 \; , \quad b_2 = \frac{2 \cdot 0 - 1 \cdot 0}{2} = 0$$

$\{a_0, a_1, b_1\} > 0 \Rightarrow$ the model does not have unstable poles.

1) Baseline models

We analyze them in time and Laplace domain.

Input used: inputs $\delta(t)$ step $\quad \mathcal{L}\begin{pmatrix} \delta(t) \\ 1(t) \end{pmatrix} = \begin{pmatrix} 1 \\ 1/s \end{pmatrix}$

pure integrater

$$G(s) = \frac{A}{s} = \frac{1}{T_i \cdot s} \; , \quad A = \frac{1}{T_i} > 0$$

T_i is integration time

Time domain behavior:

Impulse responce $\quad \underbrace{u(s)}_{\mathcal{L}(\delta(t))} \longrightarrow \boxed{A/s} \longrightarrow y(s)$

$$y(s) = G(s) \cdot u(s) = \Big|_{\delta(t)} \frac{A}{s} \cdot \underbrace{1(s)}_{\mathcal{L}(\delta(t))} = \frac{A}{s}$$

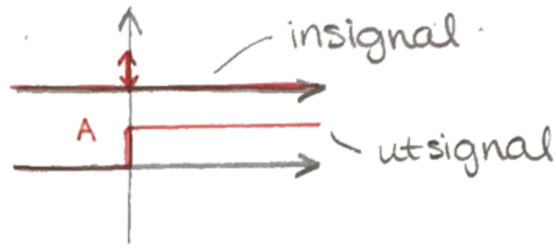
$$s\,y(s) = A\,u(s)$$

$$y(t) = A\int_0^\infty u(\tau)\,d\tau$$

$$\dot{y} = A\,u(t)$$

$$y(s)\Big|_{u(s)=1(s)} = G(s) \Big\}\ \text{impulse responce}$$

$$\mathcal{L}^{-1}(A/s) = g(t) = A\cdot 1(t)$$



BIBO since it is bounded, but (Thm 3) it isn't asymptotically stable $\lim_{t\to\infty} g(t) \neq 0$

Note, a pure integrator is often called as being stable.

<u>Step responce:</u>

$$y(s) = G(s)\,u(s)\Big|_{u(t)=1(t)} = \frac{A}{s}\cdot \underbrace{\frac{1}{s}}_{\mathcal{L}(u(t))}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{A}{s^2}\right) = At$$

Note, higher order integrals

$$G(s) = \frac{A}{s^{n_i}}, \quad n_i = \text{integr. degree}$$

First order lag

$G(s) = A/(Ts+1) \ominus$, A - (DC) gain
T - time const.

$$\ominus \left. \frac{b_0}{a_0 s + a_1} \right|_{a_0 = 1} = \frac{A/T}{s + 1/T}$$

Ex. Given T is positive, G(s) stable ?

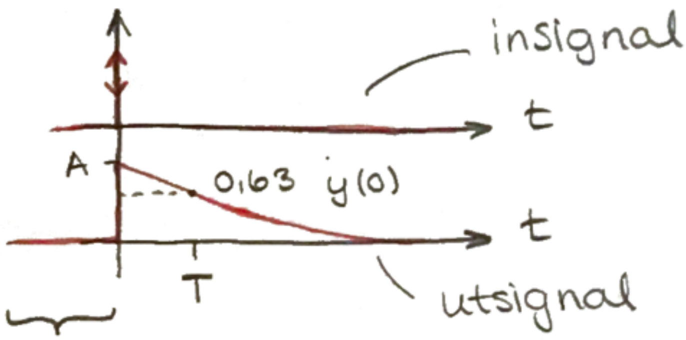
$a(s) = s + 1/T \Rightarrow p = -1/T \Rightarrow$ stable!

First order lags does not have zeros

Impulse responce

$$y(s) = G(s)u(s) = \frac{A}{Ts+1} \cdot 1(s)$$

$$y(t) = A \mathcal{L}^{-1}\left(\frac{1}{Ts+1}\right) = Ae^{-t/T}$$



$t<0$

$u(t)=0,\ g(t)=0$

$T>0$, first order lag is asymtotically stable.

ty $\lim_{t \to \infty} g(t) = 0$ ✓

Gain A (steady state / DC gain).

Ex. Suppose $u(t) = 1(t)$ — GU

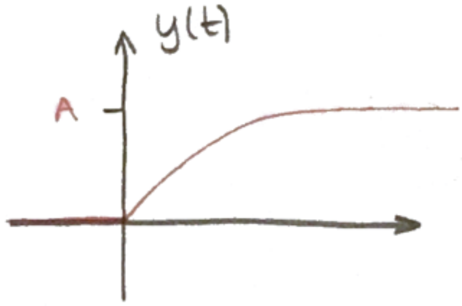
$$y_\infty = \lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,y(s) = \lim_{s\to 0} s\cdot\frac{A}{Ts+1}\cdot\frac{1}{s} = A$$

Step responce

$$y(t)\Big|_{u(t)=1(t)} = \mathcal{L}^{-1}\left(\frac{A}{Ts+1}\cdot\frac{1}{s}\right) = A\,\mathcal{L}^{-1}\left(\frac{1/T + s - s}{(s+1/T)\,s}\right) =$$

$$= A\,\mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{(s+1/T)}\right) = A\left(1-e^{-t/T}\right)$$

$y(0) = 0$, $y_\infty = A$



Second order lag

$$\frac{y(s)}{u(s)} = G(s) = \frac{b_0}{s^2+a_1 s+a_2} = \frac{A\,\omega_0^2}{s^2+2\xi\omega_0 s+\omega_0^2}$$

generic fcn.

ω_0 - natural / eigen frequency

ξ - relative damping

<u>Stability</u> (according to ξ)

<u>Case 1:</u> well damped, $\xi > 1$

$a(s) = s^2 + 2\xi\omega_0 st + \omega_0^2 \Rightarrow P_1, P_2 = -\xi\omega_0 \pm \omega_0\sqrt{\xi^2-1}$

$$\xi > \underbrace{\sqrt{\xi^2-1}}_{\text{real}}$$

$\Rightarrow$ 2 poles, both real valued and $Re(P_1, P_2) \le 0$

(stable)

Case 2: $\xi = 1$

$P_{1,2} = -\xi\omega_0$

2 poles at $-\xi\omega_0$

2 real valued poles $\longrightarrow$ stable

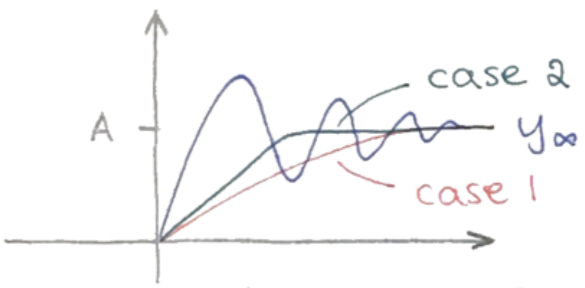
Case 3: $0 < \xi < 1$ underdamped (oscillation)

$$P_1, P_2 = \underbrace{-\xi\omega_0}_{\substack{\text{stable} \\ \text{real part}}} \pm \underbrace{i\omega_0\sqrt{1-\xi^2}}_{\text{complex}}$$

Complex poles comes with conjugates in pairs.

Case 4: $\xi < 0$ (physically not meaningful)

unstable!



step responce

ξ minskar $\longrightarrow$ oscillationer ökar

Higher order lags

$$G(s) = \frac{b_0}{s^n + \ldots + \underbrace{a_{n-1}s + \underbrace{a_n}_{\text{0 order lag}}}_{\text{first order lag}}}$$

osv.

Note: lags can be added to integrators and diffrantiators (se ex.)

$\forall$ G(s) we can split them to lags/int/diff

Time delayed model

Ex. Find transf. fcn?

$y(t) = u(t-\tau)$ - pure time delay

$G(s) = ?$

Apply Laplace Transform

$y(s) = \mathcal{L}(u(t-\tau)) = \int_0^\infty u(t-\tau)e^{-st}\,dt \circleddash$ change vars.

$(t-\tau) = T$, $t=0 = T+\tau \Rightarrow -T = \tau$

$$\circleddash \int_{-\tau}^{\infty} u(T)e^{-s(\tau+T)}\,dT = e^{-s\tau}\underbrace{\int_{-\tau}^{\infty} u(T)e^{-sT}\,dT}_{u(s)}$$

$$\Rightarrow \frac{y(s)}{u(s)} = e^{-s\tau}$$

Not a rational transfer func. hence approx.

them improper

$$e^{-s\tau} = 1 - s\tau + \frac{(s\tau)^2}{2!} + \dots$$

proper approx. $e^{-s\tau} = \dfrac{e^{-s\tau/2}}{e^{s\tau/2}} = \dfrac{1 - \frac{s\tau}{2} + \frac{(s\tau)^2}{2!} + \dots}{1 + \frac{s\tau}{2} + \frac{(s\tau)^2}{2!} + \dots}$

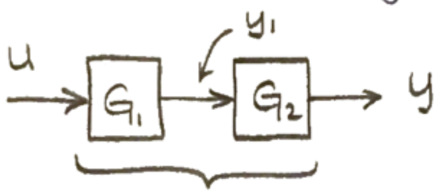
strictly proper approx:

$$e^{-s\tau} = \frac{1}{e^{s\tau}} = \frac{1}{1 + s\tau + \frac{(s\tau)^2}{2!} + \dots}$$

have to approx. to handle it!

Note, delays causes problems in frequency domain to!

Interaction of system modul.

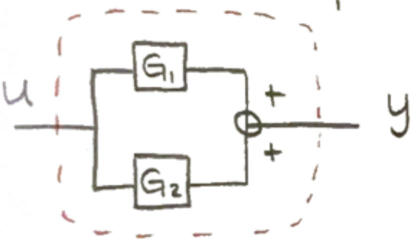
A) 

serial

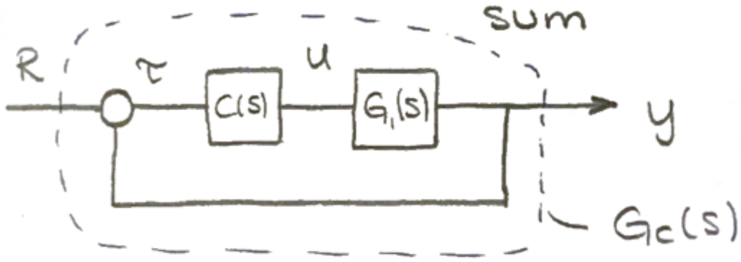
$$G(s) = y(s)/u(s)$$

$$y_1 = G_1 u$$

$$y(s) = G(s)u(s) = \underbrace{G_1 G_2}_{\text{product}} u(s)$$

B) 

$$y(s) = G_1 u + G_2 u = \underbrace{(G_1 + G_2)}_{\text{sum}} u$$

C) 

FEEDBACK

R - reference signal

E - error signal

$G_c(s)$ = closed loop transferfcn.

C(s) - controller

$G_1(s)$ - transfer fuc. model

$$G_c(s) = \frac{y(s)}{R(s)}$$

$$\underbrace{y(s) = G(s)u(s)}_{\text{plant}}, \quad \underbrace{u(s) = E(s)C(s)}_{\text{controller}}$$

$$E(s) = R - Y = R - G \cdot C \cdot E$$

$$y = GC(R - y) \Rightarrow \frac{y}{R} = G_c = \frac{GC}{1 + GC}$$

GC - loop transfer func.