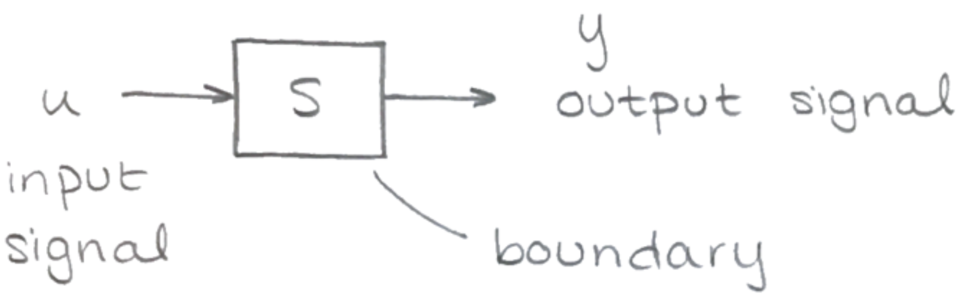
19/3-18 Föreläsning

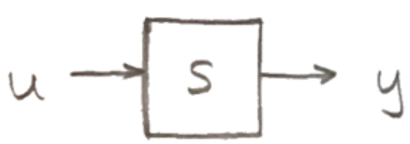
- Define a system and it's properties
- System models (mathmatical abstraction to formulate a system)

Def. of system, properties
System is an object that give outputs to inputs



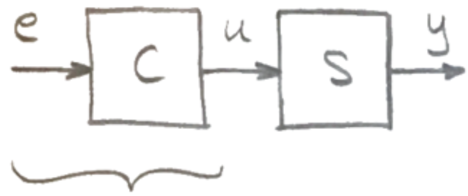
Def. Given the triplet $\{\{u, y, x\}, T, s\}$ to describe a system with input signal space u and output y with internal dynamic variable space x, over the time interval T. The system $S: \{u, x\} \times T \mapsto \{y, x\} \times T$

Ex. Heatbalance of HA4.
Input: heater's power
Output: temp HA4
Open-loop system



no coupling between $\{u, y\}$

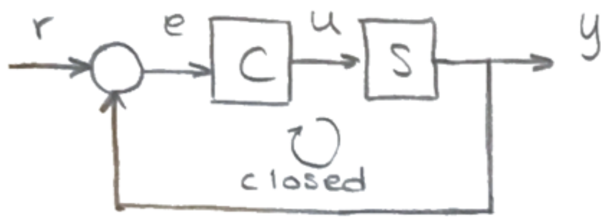
Open-loop control



controller ; no coupling between $\{e, y\}$, no feedback

examples: dishwasher

Closed-loop control system



r - reference signal

$\varphi = y - r$; output error

Controller C to be designed.

Properties of system

We narrow down the set of system descriptions to yield a "proper" model

A) Linearity: the signals superposition principle is satisfied.

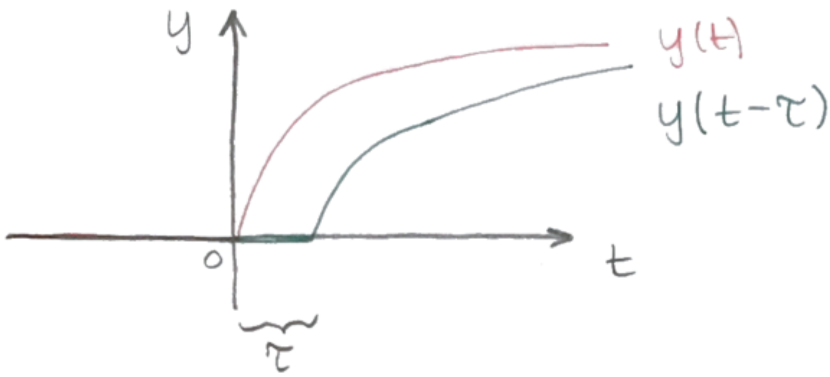
Superposition = additive + scalarable

ex. $y_1 = S(u_1)$ $\quad \alpha, \beta > 0$ scalars

$y_2 = S(u_2)$

$$y(t) = S(\alpha \cdot u_1 + \beta \cdot u_2) = \alpha y_1 + \beta y_2$$

B) Time-invariance (system prop. does not change with time, time-invariant)
S is time invariant if
$y(t) = S(u(t))$ and $\tau > 0$
$y(t-\tau) = S(u(t-\tau))$ are identic if started from the same initial condition.



c) Causality: The system is nonanticipating

The system's current output $y(t)$ does not need future input sequences.
example: Casual
$y(t) = S(u(t))$
possible depends on past and current $u(t)$.
Strictly casual
$y(t) = S(u(t-\tau))$, $\tau > 0$
Only the past input is required.
Anti-casual
$y(t) = S(u(t+\tau))$, $\tau < 0$
Only the future input is required.

D) Dynamic and static

$y(t) = S(t, u(t))$ - D

$y(t) = S(u(t))$ - S

Dynamic systems

have "memory" / inner dynamical variable

E) Finite dimensional systems

- input and output dimension;
  Single-input
  Single-output systems (SISO)
- number of inner variables is finite too.

System models

Finite dimensional, linear and time-invariant, casual dynamical system models.

$$\boxed{\sum_{i=0}^{n} a_n \cdot \frac{dy^{(n-i)}}{dt^{(n-i)}} = \sum_{j=0}^{m} b_j \cdot \frac{du^{(m-j)}}{dt^{(m-j)}}}$$

ODE's with constant coeff's. (a_i, b_j)

$0 < m, n < \infty$

(note if $m = n = 0$, static input / output relations).

Constant coeff's ensure time-invariance.

Dynamism in the model; given a set of initial conditions, with the model we can simulate the system behavior.

If we solve the ODE
TIME / LAPLACE.

TIME domain will come soon.

$y(t) = y_H(t) + y_{iH}(t)$

LAPLACE

Because all differential proplems in ODE will turn into an algebraic.

$$\mathcal{L}\left\{ \sum_{i=0}^{n} a_n \cdot \frac{dy^{(n-i)}}{dt^{(n-i)}} = \sum_{j=0}^{m} b_j \cdot \frac{du^{(m-j)}}{dt^{(m-j)}} \right\}_{y_0}$$

All initial conditions are zero; $(a_0 = 1)$

$(s^n + s^{n-1} \cdot a_1 + \ldots a_n) \cdot Y(s) =$

$(b_0 s^m + b_1 s^{m-1} + \ldots b_m) u(s)$

where $\mathcal{L}\{u(t)\} = u(s)$

$\mathcal{L}\{y(t)\} = Y(s)$

$a(s) Y(s) = b(s) u(s)$

Given a specific $u(s)$;

$Y(s) = ?$ $(y(t) = ?)$

$$G(s) = \frac{Y(s)}{u(s)} = \frac{b(s)}{a(s)}$$ (transfer function)

$Y(s) = G(s) \cdot u(s)$

$$\mathcal{L}^{-1}\{Y(s)\} = \int_{-\infty}^{\infty} g(\tau) \cdot u(t-\tau)\, d\tau$$

inverse laplacetransf.

Note, we convolute $g(t) = \mathcal{L}^{-1}\{G(s)\}$ and the input to get $y(t)$

$g(t)$ - impulse response.